$$\nabla^2 U(z) + k^2 \left|F'(z)\right|^2 U(z) = 0$$

$$\lim_{z \to \infty} |z|^{\frac{1}{2}} \left\{ \frac{\partial U(z)}{\partial n} - ik\, U(z) \right\} = 0$$

U(z) at boundary is equal to the transformed boundary value.

Since in the transformed domain the Green's function is known, we can obtain

$$U = \oint_{\text{boundary}} U_B \frac{\partial G}{\partial n}\, d\ell + k^2 \iint_{\substack{\text{exterior}\\ \text{domain}}} \left\{ \left|F'(z)\right|^2 - 1 \right\} GU\, dxdy \,.$$

Since $|z| \left\{ \left|F'(z)\right|^2 - 1 \right\}$ is bounded at infinity we have the Fredholm integral equation and U can be obtained in terms of the Fredholm series.

When the original domain and the transformed domain is geometrically similar, the kernel will be small, and the Neumann series converges. For low frequency it is convenient to transform to a circle or an ellipse for which the Green's function is known.

For high frequency, we can transform into an arbitrary smooth curve which is sufficiently similar geometrically to the original curve. Then the first term of the Neumann series will be sufficient. In this case

$$\lim_{r_2 \to \infty} \left. \frac{\partial G(r_2, r_1, \phi_1)}{\partial n_1} \right|_{\substack{r_1\\ \text{on the}\\ \text{boundary}}} \simeq \frac{e^{ir_2}}{\sqrt{r_2}} \left. \frac{\partial \psi(r_1, \phi_1)}{\partial n_1} \right|_{\substack{r_1\\ \text{on the}\\ \text{boundary}}}$$

$\dfrac{\partial \psi(r_1, \phi_1)}{\partial n_1}$ on the boundary is the normal derivative of the total field on the boundary when the boundary is smooth, a plane wave is incident, and the total field satisfies the Dirichlet boundary condition. This function can be evaluated by Fock's method.

For the high frequency case the contribution to the scattered field due to a discontinuity in the n'th derivative of the curvature can be shown to be of the order of $(1/k)^n$ by using the property that the order of the differentiability of the mapping function on the boundary is the same as the order of discontinuity of curvature.